International Journal of Advanced Research and Multidisciplinary Trends (IJARMT)
An International Open Access, Peer-Reviewed Refereed Journal
Impact Factor: 6.4 Website: https://ijarmt.com ISSN No.: 3048-9458

Theory and Mathematical Formulation of Electromagnetic Wave Propagation in One-Dimensional Photonic Crystals

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Abstract
Photonic crystals (PCs) are periodic dielectric (and sometimes metal–dielectric or metamaterialbased) structures that enable strong control over electromagnetic wave (EMW) propagation through the formation of photonic band gaps (PBGs). The theoretical foundation of PCs lies in Maxwell's equations combined with appropriate constitutive relations and boundary conditions at material interfaces. In this paper, we present a concise theory and mathematical formulation for optical wave propagation in layered media with special emphasis on one-dimensional (1-D) periodic structures. The refractive index (optical density) is introduced as a key phenomenological parameter connecting macroscopic material response to microscopic polarization mechanisms. For isotropic media, the electromagnetic fields satisfy decoupled Helmholtz-type wave equations, while for nonuniform periodic media the fields obey a master equation that leads to Bloch-mode solutions and dispersion relations within the first Brillouin zone. The transfer matrix method (TMM) is described as an efficient technique for evaluating band structure, reflectance, transmittance and defect-mode behavior in 1-D photonic crystals. The formulation is extended conceptually to anisotropic and composite media where permittivity and permeability may become tensor quantities, enabling hyperbolic dispersion and tunable optical responses. This theory provides a rigorous basis for designing photonic devices such as filters, mirrors, sensors and waveguiding structures across optical and terahertz regimes.

Keywords
Photonic crystals; Maxwell's equations; Photonic band gap; Transfer matrix method; Bloch theorem; Dispersion relation; Isotropic and anisotropic media; Effective medium theory.

Introduction

The propagation of electromagnetic waves in material media is governed by the interaction of electric and magnetic fields with the charged constituents of matter. This interaction is central to optics and photonics, where the goal is not only to understand light–matter behavior but also to engineer materials and structures capable of controlling light in a predictable and efficient way. Among such engineered media, photonic crystals (PCs) have emerged as a fundamental class of periodic optical materials that can manipulate the flow of electromagnetic energy in close analogy to how crystalline solids control electron motion through electronic band structures. The defining feature of photonic crystals is the periodic modulation of refractive index (or equivalently, dielectric permittivity), which leads to photonic band structures and the formation of forbidden frequency ranges called photonic band gaps (PBGs). Within these gaps, wave propagation is suppressed, enabling applications including highly reflecting mirrors, narrowband filters, resonant cavities, waveguides and biosensors.

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A key step in modeling photonic crystal behavior is the use of macroscopic optical parameters such as refractive index, dielectric constant, susceptibility and conductivity to describe complex microscopic interactions in an averaged form. In most optical systems of interest, the wavelength of light is much larger than the interatomic spacing, which justifies the continuum approximation. Under this assumption, the material response can be expressed through constitutive relations that connect electric displacement D\mathbf{D}D to electric field E\mathbf{E}E and magnetic induction B\mathbf{B}B to magnetic field intensity H\mathbf{H}H. For linear, homogeneous media, these relations are commonly written as D=εE\mathbf{D}=\varepsilon \mathbf{E}D=εE and B=μH\mathbf{B}=\mu \mathbf{H}B=μH, where ε\varepsilonε and μ\muμ represent permittivity and permeability, respectively. In optical frequency regimes, the polarization response of electrons particularly bound and conduction electrons dominates and absorption can be represented through complex permittivity (or a complex refractive index) whose imaginary part accounts for attenuation. The refractive index nnn, also called optical density in many contexts, is a central quantity because it directly determines phase velocity and wave propagation in a medium. For non-magnetic media, the refractive index is largely governed by permittivity, while in general form it satisfies n2=εrμrn^2=\varepsilon_r \mu_rn2=εrμr, where εr\varepsilon_rεr and μr\mu_rμr are relative permittivity and relative permeability. This relation highlights why engineered materials such as metamaterials can support unusual propagation regimes when ε\varepsilonε and μ\muμ are tailored. In particular, materials with positive refractive index (PIMs) behave conventionally, whereas negative-index materials (NIMs) can exhibit reversed refraction, backward-wave propagation and non-intuitive phase–energy relationships. The connection between refractive index and wave vector is expressed through k=(nω/c)k^\mathbf{k}=(n\omega/c)\hat{k}k=(nω/c)k^, showing that both wavelength and phase accumulation depend on the optical density and operating frequency. Wave behavior at interfaces is equally important because photonic crystals consist of repeated interfaces between materials of different refractive indices. Boundary conditions derived from Maxwell's equations enforce continuity of tangential components of E\mathbf{E}E and H\mathbf{H}H across interfaces (with corresponding conditions on D\mathbf{D}D and B\mathbf{B}B depending on charge and current distributions). These conditions lead to reflection and transmission at each layer boundary. In a periodic multilayer system, multiple reflections interfere constructively and destructively depending on optical path lengths, incidence angles, polarization (TE/TM) and refractive index contrast. When the periodicity is comparable to the wavelength, Bragg scattering becomes significant and produces photonic band gaps frequency regions in which propagating solutions are not allowed and the fields become evanescent.

For isotropic media, Maxwell's equations can be combined to yield decoupled wave equations often written in Helmholtz form for either electric or magnetic fields. However, in periodic media, permittivity varies spatially and the resulting wave equation becomes an eigenvalue problem. A widely used formulation is the "master equation" for photonic crystals, typically expressed in vector form for the magnetic field:

$$\nabla \times \left(\frac{1}{\varepsilon_r(\mathbf{r})} \nabla \times \mathbf{H}(\mathbf{r}) \right) = \left(\frac{\omega^2}{c^2} \right) \mathbf{H}(\mathbf{r}).$$

Because the medium is periodic, Bloch's theorem applies: electromagnetic modes take the form of Bloch waves characterized by a wave vector within the first Brillouin zone and eigenfrequencies form bands ωn(k)\omega_n(\mathbf{k})ωn(k). The resulting dispersion relations reveal allowed photonic bands and forbidden gaps, providing the conceptual and computational basis for designing photonic devices. For one-dimensional photonic crystals, where periodicity exists along a single axis, the transfer matrix method (TMM) offers an efficient way to compute band structure and spectral properties. In TMM, each layer is represented by a characteristic matrix connecting field amplitudes at one boundary to those at the next. Multiplying matrices across a unit cell yields a total transfer matrix from which Bloch wave number, reflectance, transmittance and defect-state resonances can be derived. Importantly, TMM accommodates oblique incidence and polarization dependence, making it suitable for real device modeling. Beyond isotropic media, many modern structures employ anisotropic, lossy, plasmabased, superconducting, or hyperbolic composite layers, where permittivity (and sometimes permeability) must be treated as tensors. In such cases, dispersion surfaces may become hyperbolic and support high-kkk modes, enabling extreme confinement and tunability. Effective medium theory further provides approximate analytical expressions for composite permittivity in multilayer metal–dielectric systems, linking optical response to filling fraction and constituent parameters. Overall, the theory and mathematical formulation of photonic crystals unite electromagnetic fundamentals with periodic-structure physics. This framework enables systematic prediction of band gaps, field localization and spectral signatures and it supports a wide range of applications from filtering and sensing to integrated photonic circuitry.

Isotropic and anisotropic medium

It was also seen earlier that the optical density of the material is in dependence on two physical properties of the materials; i.e, electric permittivity and magnetic permeability. When EMWs are interacted with these materials, such interaction is related to these parameters with the polarizations of material. Nevertheless, ε or μ is tensor quantity. Therefore, the materials are two types: (i) isotropic material when ε and μ are independent of direction whereas (ii) anisotropic material when ε and μ is direction dependent. The isotropic and anisotropic materials follow cubic symmetry mechanism and inhomogeneous medium follow composite asymmetry, respectively. It has been observed from the time of Maxwell that transport properties of randomly inhomogeneous materials have been of great intrigue to the researchers. In the today 's world of science and technology, novel materials practicing the novel material demand of high-speed information are characterized with unique optical properties which the already existing materials can not possess. The scientists always manage to exhaust the amount of research on manufacturing or developing materials with such unique properties or modify the available materials and the present materials are fabricated by the available technique, e.g. the preparation of thin films in nanoscience and nanotechnology and so on.

It is to state of the several approaches that provide the best optical properties of materials called composite materials, the properties of composite thin films are usually changed in a more convenient way and thus it is easy to implement. Evaporation, sputtering and ion beams assisted depositions are also the techniques of modification or deposition of thin films which have been proven to be quite successful in *preparing composite or inhomogeneous or anisotropic dielectric thin films*. It is demonstrated that the manufacturing of thin films using the above techniques is helpful in the fabrications of optical thin film devices. Dielectrics have been co deposited with different metals to form Cermets films which were then applied in devices for conversion of solar energy. Extensive research on both dielectric-dielectric and metal - dielectric composite thin films has been done in the aspects of optical properties in the near infrared or solar regions [1-4].

The most astonishing fact about the thin metallic films is that the metallic films show different optical properties from those of just the bulk metal. These films show very selective absorption and the optical properties strongly depend on the film structure, for example their thickness. The electron-microscopic results suggested that actual thin films are not parallel-sided homogeneous slabs, but they are films having some in-homogeneity like unevenness or some cracks or particles isolated from each other. The experimental results indicate that the very thin films of silver become in resonance absorption and have the peaks occur at about 435µm or at still longer wavelength when the films are heated. The films consist of many small particles of silver [5]. Another result of the experiment indicated that thin metallic films could be considered as an aggregate of small rotational ellipsoids in two dimensions. Resonating of the free electron gas bound within an ellipsoid at some frequency depends on the shape of the ellipsoids [6]. These results reveal that the thin films of the materials have different optical properties or different optics of material due to thickness. The interaction of light or EMW with thin film material can be analyzed by studying the EMWs in thin film using MEs.

EMW s in isotropic medium

An isotropic material is that in which the optical properties are independent of the polarization state of an EMW when the EMW passes through the material [7]. Some constitutional relations for isotropic media hold true and the MEs can be solved easily.

Fields and waves in isotropic medium

The EF of the EMW interacting with an isotropic medium induces polarisation and it is the parallel component of the EF that determines the strength of the induced polarisation. From [8] it is demonstrated that the induced polarisation is proportional to the EF:

$$\vec{p} = \varepsilon_0 3\vec{E} \quad (2.5)$$

Where $\vec{p}$= polarization,$\vec{E}$ =EF of wave, χ =electric susceptibility and ε_0 =electric permittivity of free space.

As we know, the constitutional relation for displaced EF and induced polarization and the relation are given by:

$\vec{D} = \varepsilon_0\vec{E} + \vec{P}$ (2.6)

Where $\vec{D}$ = dielectric displacement vector, $\vec{P}$ = induced polarization, $\vec{E}$ = EF of wave.
On substituting polarization form Eq. (2.5) in Eq. (2.6), we get,

$\vec{D} = \varepsilon_0(1 + 3)\vec{E}$ (2.7)

$(1+\chi)\varepsilon_r$, called relative permittivity of the material, then Eq. (2.7) becomes:

$\vec{D} = \varepsilon_0\varepsilon_r\vec{E}$ (2.8)

Generally, the optical constant of the material is given by:

$$\varepsilon_r=\varepsilon'-i\varepsilon'' \quad (2.9)$$

where ε′ real electric permittivity and ε″ imaginary electric permittivity.
This relative electric permittivity is called the square optics of material or square of optical density or square refractive index. The optical density of the material is given by:

$n= \sqrt{\varepsilon_r} = \sqrt{\varepsilon' - \varepsilon''}$ (2.10)

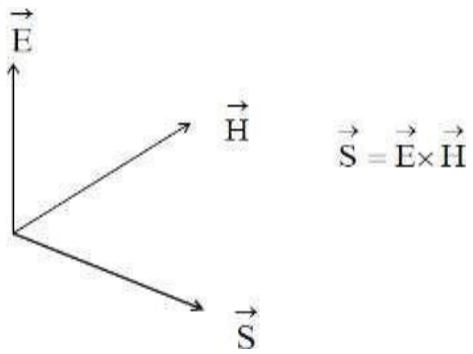

Figure 2.1: The EF ($\vec{E}$), magnetic field intensity ($\vec{H}$) and Poynting vector ($\vec{S}$) in isotropic medium.

The Figure 2.1 explains the direction of energy propagation with the coupling field waves. The local direction of $\vec{S}$ is also called the ray direction. Only for the wave front is the ray direction which is the same at all points on the phase fronts. The best way to understand the properties of an optical system is to find out when a ray of light enters in the medium. It is necessary to understand the direction of light as it encounters medium as well as the intensity of light when it crosses an interface [8].

MEs for material

The EMW propagation in a material medium is started by the four most fundamental equations of electrodynamics, which are known as the MEs:

$\vec{A}.\vec{D} = \rho$ (2.12)

$\vec{A}.\vec{B} = 0$ (2.13)

$\vec{A} \times \vec{E} = -6\vec{B}$ (2.14)

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—
6t

(2.15)

A→ × H→ = →J
6D→
6t

Where, E and H denotes electromagnetic field and EF, respectively. EMW field in the material is termed by these two fields D and B→; where D shows the DD vector and B→ is the magnetic induction. These four field vectors are related with the constituent relations, which include the effect of the wave field on materials. The quantities ρ and →J denotes density of electric charge and density of current associated with wave field generatorsE and H→. Using these equations govern the EMW field in the medium. MEs cannot be derived exclusively unless the connection between B & H and E & D are known to get a unique purpose of the wave field vectors because these variable contains the eight scalar equations which relates to the twelve variables, three for each four vectors E, H, D and B→. The materials constituent equations are:

$$\vec{D} = \varepsilon_0\vec{E}+\vec{P}=\varepsilon_0(1+\chi_e)\vec{E}=\varepsilon\vec{E} \quad (2.16)$$

$$\vec{B}=\mu\vec{H}=\mu_0(\vec{H} + \vec{M})=\mu_0(1+\chi_m)\vec{H} \quad (2.17)$$

Here, ε and μ are tensors of rank two; P→ and M→ are electric and magnetic polarization respectively. When electrons, whose motion is not disturbed from 0 permittivity, are disturbed by electric wave field, electric dipole polarisation P per unit volume is created. Magnetic field also induces a magnetization M per unit volume in the materials where permeability is not exactly 0. For permeability, constant 0 is 4 x 107 H/m. Both the and tensors reduce to scalars in an isotropic or linear medium. We assume that the field forces have no effect on the amounts and. However, for the anisotropic medium, the dependency of quantities and on μ on E→ and H→ must be on included for the sufficiently strong field.

The MEs have coupled field wave equations. The Eq. (2.14) in the MEs for the isotropic medium is given below:

A× E = − 6B = −μ 6H→ → 6t 6t

Now, we have considered that a plane wave ofE andB is incident on the isotropic medium. The E→ (EF) and the H(magnetic field) a multiplication of a function of the angular form over time with a function of the location of the wave vector as:

$$\vec{E}(r,t) = \vec{E}_0 e^{-i\omega t+i\vec{k}.\vec{r}} \quad \text{and} \quad \vec{H}(r,t) = \vec{H}_0 e^{-i\omega t+i\vec{k}.\vec{r}} \quad (2.19)$$

Where $\vec{E}_0$ and $\vec{H}_0$ are the initial electric and magnetic fields.

Using Eq. (2.19), the derivatives of MEs become:

6 ① −iω (2.20)

—
6t

and [6] ① ik A ① −ik (2.21)

—
6x

Using Eq. (2.19), the Eq. (2.18) becomes:

$\vec{A} \times \vec{E} = -im\vec{B} = -im\mu\vec{H}$ (2.22)

where μ = magnetic permeability, ω= frequency of time harmonic fields an $\mu=\mu_0(1 + \chi_m)$. Similarly, from the MEs , the Eq. (2.15) takes the form:

$\vec{A} \times \vec{H} =$ →J + (2.23)

→ →
D→
6
6t

Using Eq. (2.19), the Eq. (2.23) becomes:

$\vec{A} \times \vec{H} = J - im\vec{E}$ (2.24)

where J = total current density and $\varepsilon = \varepsilon_0(1+\chi_c)$.

From MEs, the Eq. (2.12) gives the electric permittivity:

$\vec{A}.\vec{E} = \frac{p}{s}$

Where, ρ = total charge density and $\varepsilon = \varepsilon_0(1+\chi_c)$.

Similarly, from Maxwell's equation, Eq.(2.17), the equation can be simplified:

$\vec{A}.\vec{B} = \vec{A}.(\mu\vec{H}) = 0$ (2.26)

Where $\mu = \mu_0(1+\chi_m)$ and $\vec{B} = \mu\vec{H}$||

Calculation of optical properties of PC The two ways of solving the Maxwell's equation are (i) scalar form and (ii) vector form. According to the review of books and journals for the periodic structures/PC s, there are six main numerical methods used to simulate the periodic structures optical properties: (1) PWEM [14], (2) FDTD method [15], (3) FEM [16], (4) TMM [17], (5) RTSM [18] or a set of considered spheres [19] and (6) DGM [20]. Above methods are used to calculate with maximum efficiency for the optical property of PC. These methods give results with full accuracy and the good agreement with practical results. Tackling of problems through the different methods that are to be used by the PC s is known to be the one that leads the selection. The last part of the methods, such as PWEM, FDTD, FEM, TMM, all of these

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have the advantage to model doped as well as undoped crystals [15–17] and 2D (2D) PC s are the parallel cylinder form while; 3D PC s are spherical form [18, 19]. Step (1), (4) and (6) are also valid for infinite crystals [15, 16, 20], whereas step (5) is specific to finite structures. Finally, using PWEM, FDTD, FEM, TMM, RTSM and DGM, the defect structures are analyzed using a super cell. Methods (2), (3) and (5) can however be used with a finite, structure with a single defect.

Plane wave expansion method

In order to determine the band structure of a periodic structure, PWEM is a simple and straightforward procedure. The imperfect infinite PC will be analysed as a super-cell. This technique is widely used for calculating several findings [21–23] while assessing the band structure of materials. The method's memory capacity is limited since it relies on a predetermined group of plane waves to grow the field. When the periodic structure of the PC is broken, this set grows in size.

FDTD method

This describes Maxwell’s equations in time domain. According to these findings, these systems [24–26] have very desirable behaviour in experimental settings. Sometimes, transmission ratio of optical materials is adopted to determine the electromagnetic mode of the defect mode in FEM. EMW pulse is incident to the material where the EMW signal is detected and the permeability of the periodic structure is calculated. FDTD technique may be used to model the crystals having either the internal or external EMW sources. For this reason, a complete practical setup with a periodic structure can be simulated using this method. This approach may be used to simulate the optical properties of the PC s. The main drawbacks of the FDTD are: (1) the treatment of certain materials, e.g., thin wires, is not accurate and (2) the size of memory needed to compute large crystals. This approach has many benefits, including the ability to accurately model anisotropic or non linear materials [16].

Conclusion

This paper presented the theoretical and mathematical basis for electromagnetic wave propagation in photonic crystal periodic structures, emphasizing one-dimensional layered systems. Starting from Maxwell’s equations and constitutive relations, optical density (refractive index) was established as a key macroscopic descriptor of light–matter interaction. By applying interface boundary conditions and periodicity principles, photonic band gaps emerge naturally as forbidden propagation regions arising from Bragg interference in refractive-index-modulated media. The master equation formulation provides an eigenvalue framework for deriving photonic band structures, while Bloch’s theorem explains the band formation within the first Brillouin zone. For 1-D photonic crystals, the transfer matrix method was highlighted as a practical and powerful approach for computing dispersion relations, reflectance, transmittance and defect-mode resonances under different incidence and polarization conditions. The discussion also connected isotropic modeling to anisotropic and composite media, where tensor permittivity and effective medium theory enable advanced regimes such as hyperbolic dispersion and tunable photonic behavior. Collectively, these formulations offer a robust foundation for designing and optimizing photonic devices including mirrors, filters, waveguides and high-sensitivity sensors.

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