Design And Development Of Basic Mathematics Modules Based On Professional Competency Requirements

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ABSTRACT

The proposed paper suggests a competency-based model of systematic designing and development of Basic Mathematics modules based on the requirements of the professional skills. Unlike the traditional content-based curricula, the approach proposed puts a strong emphasis on measurable competencies, formal mathematical rigor, and mastery with applications. This is a formalization of fundamental mathematical domains, i.e. Algebra, Calculus, Linear Algebra, Probability, and Optimization, into coherent modules in an explicit competency-mapping mechanism. The modules are strictly developed involving accurate definition, suggested theorems, full proofs and systematic formulations of the problems and providing conceptual depth and relevance. The framework provides a clear correspondence between the learning outcomes and the industry expectations and maintaining the integrity of mathematical underpinnings. In addition, sample modules with proofs and examples solved are to be offered to show the implementation and prove the effectiveness of the suggested model to promote analytical reason and computational skills as well as profession-specific problem-solving abilities.

Keywords: Competency-Based Education, Mathematics Modules, Professional Skills, Modular Learning, Applied Mathematics

1. INTRODUCTION

The contemporary tendencies in the design of professional education curricula are increasingly moving toward a competency-based approach. This is a method in which the content of the curriculum is systematically connected with clearly defined professional competencies and measurable learning outcomes. As a result of this shifting paradigm, education is no longer primarily focused on the transmission of theoretical knowledge; rather, it is based on the development of practical abilities that students are able to effectively apply in situations that are actually occurring in the real world. Within the confines of this framework, mathematics is undergoing a process of redefining itself from an abstract and solitary field of study to a fundamental foundational instrument that supports a wide variety of professional competencies. Analytical reasoning, organized problem-solving, quantitative decision-making, and mathematical modeling are all vital in modern domains such as engineering, data science, management, economics, and applied sciences. It plays a critical role in strengthening these skills, which are all essential in these fields.

In addition, the incorporation of mathematics into professional education places an emphasis on the relevance of mathematics in the resolution of complicated problems that include multiple

disciplines. This serves to make the learning experience more relevant and meaningful for students. This shift helps students to comprehend not only the theoretical underpinnings of mathematics but also the practical consequences and implementations of those concepts in a variety of contextual settings. As a consequence of this, students acquire the skills necessary to evaluate data, design models, optimize solutions, and make decisions based on logical and quantitative analysis that are informed about the situation. Additionally, the competency-focused approach encourages continual evaluation and skill validation, which guarantees that students will attain mastery in both conceptual comprehension and practical execution. As a consequence of this, mathematics education transforms into a dynamic and outcome-driven process that provides students with the tools they need to successfully adapt to fast changing professional contexts and effectively contribute to innovation and problem-solving in their respective disciplines.

Let

$$C = \{c_1, c_2, \ldots, c_n\}$$

denote the finite set of professional competencies, where each element c_i represents a distinct skill requirement such as quantitative reasoning, algorithmic thinking, statistical analysis, optimization ability, or modeling proficiency. These competencies are typically derived from accreditation frameworks, industry standards, and employability benchmarks.

Similarly, let

$$M = \{m_1, m_2, \ldots, m_k\}$$

represent the set of Basic Mathematics modules, where each module m_i encapsulates a coherent mathematical unit comprising definitions, theorems, proofs, computational techniques, and applied problem-solving exercises. Examples include Algebraic Methods, Differential Calculus, Linear Algebra, Probability Theory, and Optimization.

The central design objective is to construct a mapping

$$f: M \rightarrow 2^C$$

where 2^C denotes the power set of C. For each module m_i, the image $f(m_i) \subseteq C$ specifies the subset of professional competencies supported or developed through that module. This formulation allows a single module to contribute simultaneously to multiple competencies, reflecting the interdisciplinary nature of mathematical learning.

A fundamental requirement of the proposed framework is the competency-coverage condition:

$$\forall m_i \in M, f(m_i) \neq \emptyset$$

which ensures that every mathematical module has explicit professional relevance. In other words, no module exists in isolation from practical skill development; each contributes directly to at least one targeted competency. This constraint eliminates redundant or purely content-driven instruction and promotes outcome-oriented curriculum design.

Furthermore, global curriculum adequacy is achieved when the collective modules satisfy:

$$\bigcup_{i=1}^{k} f(m_i) = C$$

Connoting that the whole competency set is covered by conglomeration of all modules. This ensures that professional requirements are fully covered and competency lapses in graduate preparation are avoided.

Mathematically speaking, this formalization makes curriculum design into a structured optimization problem: given mathematical modules, how to choose and arrange them to ensure that professional competencies are met, and to reduce the amount of redundancy and maximize the amount of learning. The modules are also internally organized with rigorous mathematical elements like definitions, theorems, proofs, and solved problems hence maintaining academic depth as well as maintaining applicability.

The given mapping framework, therefore, offers the mathematically founded approach to the design of the Basic Mathematics curricula that would be both rigorous and modular, as well as, in association with these two aspects, also professionally aligned. It helps to bridge the historical gap between abstract teaching of mathematics and competency-based teaching and provides a model that can be scaled with changes to workforce requirements.

2. REVIEW OF LITERATURE

Taufikurrahman, Budiyono, and Slamet (2021) designed a mathematics course based on meaningful learning concepts whereby the instructional resource materials were systematically structured to relate abstract mathematics knowledge with what learners already knew and real life situations. Their research showed that the modules that were implemented contributed greatly to the conceptual knowledge and learning experience of the students. The authors had found that active knowledge construction and increased cognitive processing were facilitated with the help of structured learning activities, guided reflection and contextual examples. Their results indicated that meaningful learning-based modules helped not only in procedural fluency but also conceptual clarity thus substantiating the relevance of the well-constructed instructional design in mathematics learning.

Dewimarni, Rizalina, and Erdriani (2024) concentrated on the creation of Basic Mathematics modules specifically based on the professional competencies, with the focus on outcome-driven education and learning with the focus on employability. In their study, they combined analytical reasoning, problem solving abilities with applied mathematical thinking into modular teaching. The results showed that competency-based modules had great positive impact on the readiness of learners to professional activities as they enhanced higher-order cognitive abilities and practical application skills. The authors concluded that systematic mapping of mathematical content and professional competencies was more relevant to the curriculum, helped learners to be prepared to the challenges of workplaces, and promoted the achievement of quantifiable learning outcomes, which made competency-based curriculum models worth consideration.

Putri, Ferdianto, and Fauji (2020) developed a mathematics instructional module digital, based on mathematical communication in the subjects of relations and functions. Their analysis portrayed that digital interactive learning conditions enhanced students to communicate mathematical concepts symbolically, verbally, and graphically. The findings revealed that there were significant improvements in communication skills, conceptual knowledge, and

representational fluency of students. The authors emphasized that modular instruction that was supported by technology allowed learners to gain independence and interactivity and develop higher-order mathematical abilities, in particular, to interpret, model, and describe mathematical relationships.

Rochsun and Agustin (2020) created an electronic mathematics unit that is grounded on contextual problem solving strategies and integrates real-life applications into the digital platform. Their study showed that contextualized e- modules enhanced student motivation, promoted self-learning and conceptual understanding. The authors also noted that when the real-life problems were exposed, the learners became active and enhanced their capability to apply mathematical knowledge to the real-life situations. Their results highlighted the strength of the contextual learning scenarios to narrow the gap between theoretical learning and practical problem solving.

3.   COMPETENCY–MATHEMATICS MAPPING MODEL

To formalize the correspondence between professional competency requirements and Basic Mathematics instructions this paper presents a set theory based competency-module mapping framework to be used to formalize the correspondence between the two. This framework gives a methodological process of making sure that each instructional module makes a difference towards employability-oriented results.

Definition 3.1 (Competency Mapping Function)

Let

$$M = \{m_1, m_2, \ldots, m_k\}$$

denote the set of mathematics modules and

$$C = \{c_1, c_2, \ldots, c_n\}$$

denote the set of professional competencies.

A **competency mapping function** is defined as

$$f: M \to 2^C$$

such that, for each module m_i,

$$f(m_i) = \{c_j \mid m_i \text{ contributes to competency } c_j\}.$$

Here, 2^C represents the power set of C, allowing each module to be associated with multiple competencies. This reflects the multifaceted nature of mathematical learning, where a single topic (for example, linear algebra) may simultaneously develop analytical reasoning, computational proficiency, and modeling capability.

This definition creates a formal connection between what is on the curriculum and what is achieved by the professionals. The mapping does not view the mathematics modules as separated academic entities but explicitly outlines the competencies that are developed during each of the modules, thus allowing the planning of the curriculum, aligning of the assessment and evaluation of the outcomes.

Definition 3.2 (Module Completeness)

A set of modules Mis said to be **complete with respect to competencies** if

$$\bigcup_{i=1}^{k} f(m_i) = C.$$

It is claimed in this condition that the sum of the set of competencies of all the modules would be the whole set of competencies 0. Practically, completeness of modules ensures that no professional competency is left out in the curriculum. At least one mathematical module supports each of the necessary skills.

Module completeness will thus act as a universal standard of quality of curriculum design to cover all professional needs holistically and avoid instances of gaps in graduate preparedness.

Theorem 3.1 (Coverage Theorem)

If module completeness holds, then every professional competency is addressed by at least one mathematical module.

Proof

From Definition 2.2, module completeness implies that

$$\bigcup_{i=1}^{k} f(m_i) = C.$$

By the definition of set union, this equality means that for any competency $c \in C$, there exists at least one index $i \in \{1, 2, \dots, k\}$ such that

$$c \in f(m_i).$$

Equivalently, every competency in C appears in the competency set associated with at least one module m_i. Hence, each professional competency is explicitly supported by some mathematical module.

Therefore, the curriculum achieves full competency coverage.

4. STRUCTURE OF BASIC MATHEMATICS MODULES

Let each module be represented as:

$$m_i = (D_i, T_i, P_i, S_i)$$

where:

- D_i: Definitions
- T_i: Theorems
- P_i: Proofs
- S_i: Solved Problems

Module 1: Algebraic Foundations

Definition 4.1

A linear equation in one variable is:

$$ax + b = 0, a \neq 0$$

Solution:

$$x = -\frac{b}{a}$$

Theorem 4.1 (Quadratic Formula)

For

$$ax^2 + bx + c = 0, a \neq 0$$

solutions are:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Proof

Divide by a:

$$x^2 + \frac{b}{a}x + \frac{c}{a} = 0$$

Complete the square:

$$x^2 + \frac{b}{a}x = -\frac{c}{a}$$

$$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2}$$

Taking square roots:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Solved Example 4.1

Solve:

$$2x^2 - 3x - 5 = 0$$

$$x = \frac{3 \pm \sqrt{9 + 40}}{4} = \frac{3 \pm 7}{4}$$

$$x = \frac{5}{2}, -1$$

Module 2: Differential Calculus

Definition 4.2

Derivative of $f(x)$:

$$f'(x) = \lim_{h \to 0} \frac{f(x + h) - f(x)}{h}$$

Theorem 4.2 (Power Rule)

If

$$f(x) = x^n$$

then

$$f'(x) = nx^{n-1}$$

Proof

Using binomial expansion:

$$(x + h)^n = x^n + nx^{n-1}h + O(h^2)$$

Substitute:

$$f'(x) = \lim_{h \to 0} \frac{nx^{n-1}h + O(h^2)}{h} = nx^{n-1}$$

Solved Example 4.2

Find derivative of:

$$f(x) = 3x^4 - 2x^2 + 7$$
$$f'(x) = 12x^3 - 4x$$

Module 3: Linear Algebra

Definition 4.3

Matrix multiplication:

$$(AB)_{ij} = \sum_{k=1}^{n} a_{ik} b_{kj}$$

Theorem 4.3

A square matrix A is invertible iff:

$$\det(A) \neq 0$$

Proof

If $\det(A) = 0$, rows are linearly dependent $\Rightarrow$ no inverse exists.
If $\det(A) \neq 0$, adjugate gives:

$$A^{-1} = \frac{1}{\det(A)} \operatorname{adj}(A)$$

Solved Example 4.3

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$
$$\det(A) = 1(4) - 2(3) = -2 \neq 0$$

Thus invertible.

Module 4: Probability and Statistics

Definition 4.4

Probability:

$$P(A) = \frac{|A|}{|S|}$$

Theorem 4.4 (Bayes' Theorem)

$$P(A \mid B) = \frac{P(B \mid A)P(A)}{P(B)}$$

Proof

From conditional probability:

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$

and

$$P(A \cap B) = P(B \mid A)P(A)$$

Solved Example 4.4

If

$$P(A) = 0.4,\ P(B \mid A) = 0.5,\ P(B) = 0.2$$

then

$$P(A \mid B) = \frac{0.5 \times 0.4}{0.2} = 1$$

Module 5: Optimization

Theorem 4.5 (Necessary Condition for Extremum)

If fhas local extremum at x = a, then:

$$f'(a) = 0$$

Proof

At extremum slope changes sign $\Rightarrow$ derivative vanishes.

Solved Example 4.5

Maximize:

$$f(x) = x(10 - x)$$
$$f'(x) = 10 - 2x = 0 \Rightarrow x = 5$$

Maximum value:

$$f(5) = 25$$

5. COMPETENCY ASSESSMENT MODEL

Competency assessment plays a pivotal role in evaluating learner performance by systematically integrating knowledge acquisition, skill development, and application capabilities across multiple learning modules. In contemporary outcome-based education frameworks, learner achievement is no longer viewed as a single examination score but as a composite measure derived from diverse instructional components. Accordingly, this section presents a competency assessment model that represents learner performance through a multidimensional score vector and computes overall achievement using weighted aggregation. The proposed framework captures both individual module contributions and their relative pedagogical importance, thereby providing a comprehensive, quantitative, and scalable approach for measuring learner competency. This model supports transparent evaluation, curriculum alignment, and data-driven decision-making in modern educational environments. Let the learner score vector be defined as

$$L = (l_1, l_2, \ldots, l_k),$$

where each component l_i represents the learner's performance score in the ith module (or learning unit), and kdenotes the total number of modules considered in the curriculum.

Since not all modules contribute equally to overall competency, a weighting factor is assigned to each module. Let

$$w_i, i = 1, 2, \ldots, k$$

denote the module weight, reflecting the relative importance or credit value of the ith module. Typically, these weights are normalized so that

$$\sum_{i=1}^{k} w_i = 1,$$

ensuring that the final achievement score lies on the same scale as the individual module scores. The overall competency achievement Ais then modeled as a weighted linear combination of the learner's module scores:

$$A = \sum_{i=1}^{k} w_i \, l_i.$$

6. CONCLUSION

The competency-based architecture of mathematics modules that has been offered establishes a highly structured and systematic integration of fundamental mathematical theory with the acquisition of vital professional skills. This integration guarantees that learning is both conceptually rigorous and practically applicable. Each module is organized through precise definitions, formally stated theorems, logically developed proofs, and well-selected solved examples that collectively promote deep conceptual understanding along with strong problem-solving abilities. The framework is carefully designed around clearly defined competencies, and each module is organized in this manner. Mathematics education is shifted from a typical content-driven approach to an outcome-oriented paradigm as a result of this dual emphasis. In this paradigm, the focus is on what learners are able to grasp, apply, and demonstrate. As a consequence of this, students acquire abilities that are applicable in a variety of contexts, including analytical thinking, computational proficiency, logical reasoning, and mathematical modeling. These skills are all closely matched with the changing requirements of the modern workforce. In addition, the explicit mapping between modules and competencies guarantees that the curriculum is coherent, transparent, and accountable. This provides educators with the capacity to properly evaluate the outcomes of learning and to make continual adjustments. Because of the modular nature of the framework, it is also flexible and adaptable, which makes it suited for integration across a variety of interdisciplinary disciplines, including engineering, data science, economics, and other professional fields. This competency-based approach develops a scalable and dynamic learning ecosystem that bridges the gap between theoretical knowledge and real-world professional practice. It does this by combining academic rigor with practical applicability, which is a combination of academic achievement and practical application.

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